On Existence of Truthful Fair Cake Cutting Mechanisms

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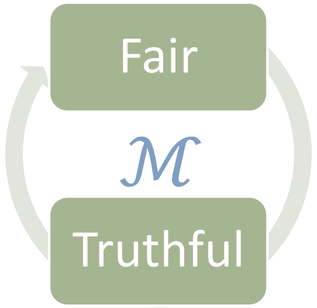
CAKE CUTTING PROBLEM

- How can we divide a divisible heterogeneous good (e.g., cake) fairly among agents?
- Divisible:
  - cake, time, land, computation resources, etc.
- Heterogeneous:
  - different agents have different preferences on different parts of the cake
- Fair:
  - Envy-free: no agent envies the allocation of any other agent
  - Proportional: each agent receives at least his/her average share

Credit: Alicia Kubista / Andrij Borys Associates
https://cacm.acm.org/magazines/2013/7/165476-cake-cutting/fulltext
Ariel Procaccia. Cake Cutting: Not Just Child's Play

MODEL

- Cake: interval $[0,1]$, allocated to n agents
- Allocation: $A = \{A_1, \dots, A_n\}$ with $A_i \cap A_j = \emptyset$
- Value Density Function $f_i: [0,1] \to \mathbb{R}_{\geq 0}$ for each agent i:
  - Agent i's value on $S \subseteq [0,1]$ is then
$$v_i(S) = \int_S f_i(x)dx$$
- Mechanism: $\mathcal{M}: (f_1, \dots, f_n) \to (A_1, \dots, A_n)$
  - Deterministic
  - Direct-revelation
- Envy-Freeness: $v_i(A_i) \geq v_i(A_j)$ for any pair of agents i and j
- Proportionality: $v_i(A_i) \geq \frac{1}{n} v_i([0,1])$ for any agent i



GAME THEORETICAL VIEW

- Agents may lie and misreport their valuations if beneficial! → Game Theory
- Truthful mechanism: mechanism under which reporting truthfully is a dominant strategy.

OBJECTIVE

A fair (envy-free or proportional) and truthful mechanism $\mathcal{M}$?

IMPOSSIBILITY RESULT

- There does not exist a mechanism that is truthful and proportional.
- The impossibility result even holds under the following settings:
  - There are only two agents.
  - Discarding some parts of the cake is allowed.
  - Agents' valuations on the cake are piecewise-constant and always positive.

RELAXING TRUTHFULNESS

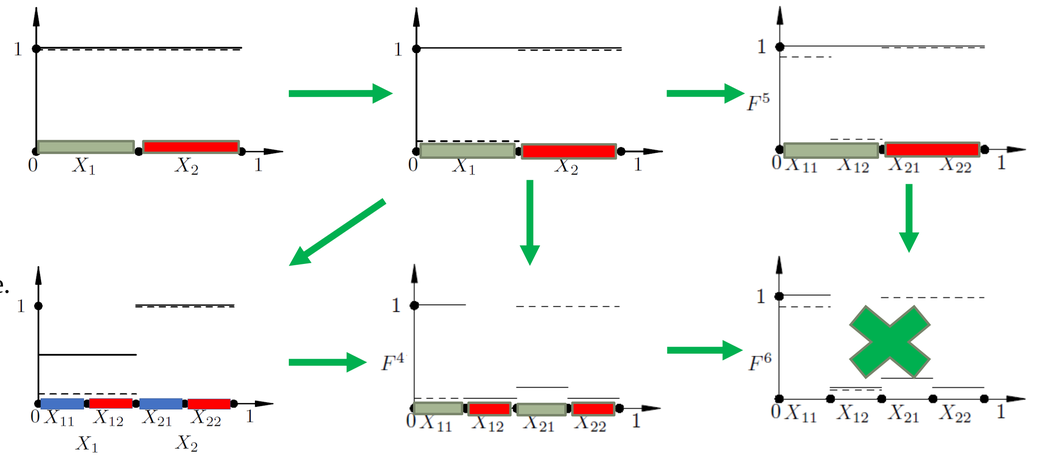
- "Risk-averse truthful" Motivated by "I-cut-you-choose"
- $\mathcal{M}$ is risk-averse truthful if
  - either misreporting f_i is non-beneficial
  - or there is a chance $\mathcal{M}$ allocate A_i to agent i with $v_i(A_i) < \frac{1}{n} v_i([0,1])$

Results:

- A variant of Dubins-Spanier's moving knife procedure
  - Proportional
  - Risk-averse truthful
  - Output allocations with connected pieces
- A simple mechanism
  - Envy-free
  - Risk-averse truthful

FUTURE WORK

- Does there exist $\alpha > 0$ such that there exists a truthful, α-approximately proportional mechanism?
- Or, does there exist a truthful mechanism that is non-oblivious to each agent's value density function?
- Or even more general, what a truthful mechanism can do?



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