Algorithmic Information Design for Singleton Congestion Games

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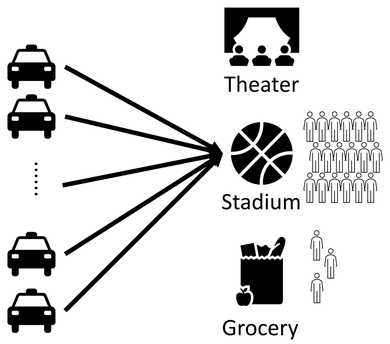
Singleton Congestion Games

- Singleton Congestion game is a fundamental class of games with negative externality.

Agents (drivers) $[N] = \{1, \cdots, N\}$

Resource (locations with orders) $[R] = \{1, \cdots, R\}$

Non-decreasing Congestion functions (orders each agent receives) $c_r\colon [N] \to \mathbb{R}^+$



$c_{theater}(n) = 2$

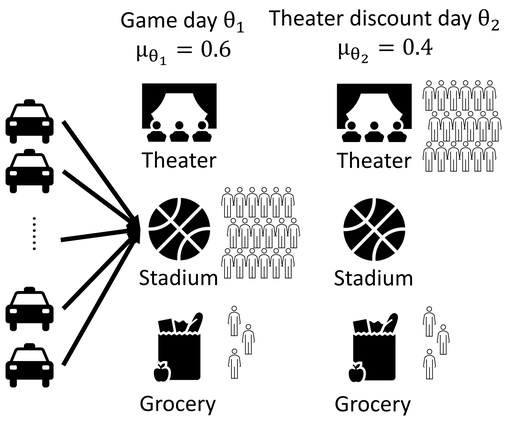
$c_{stadium}(n) = \begin{cases} 0, \text{if } n < N \\ 1, \text{if } n = N \end{cases}$

$c_{grocery}(n) = 1 + \epsilon$

Each agent selects a single resource a_i^* that minimizes congestion from his available actions $A_i \subseteq [R]$

Singleton Congestion Games w. Uncertainty

The congestion functions c_r depends on a common random state of nature θ drawn from discrete support Θ with prior distribution $\mu \in \Delta_\Theta = \{\mathbf{p} \in \mathbb{R}_+^\Theta : \sum_{\theta \in \Theta} p_\theta = 1\}$



$c_{theater}(n) = \begin{cases} 1.2, \text{if } n < N \\ 1.6, \text{if } n = N \end{cases}$

$c_{stadium}(n) = \begin{cases} 0.8, \text{if } n < N \\ 1.4, \text{if } n = N \end{cases}$

$c_{grocery}(n) = 1 + \epsilon$

Agents will choose the best response based on the prior μ:

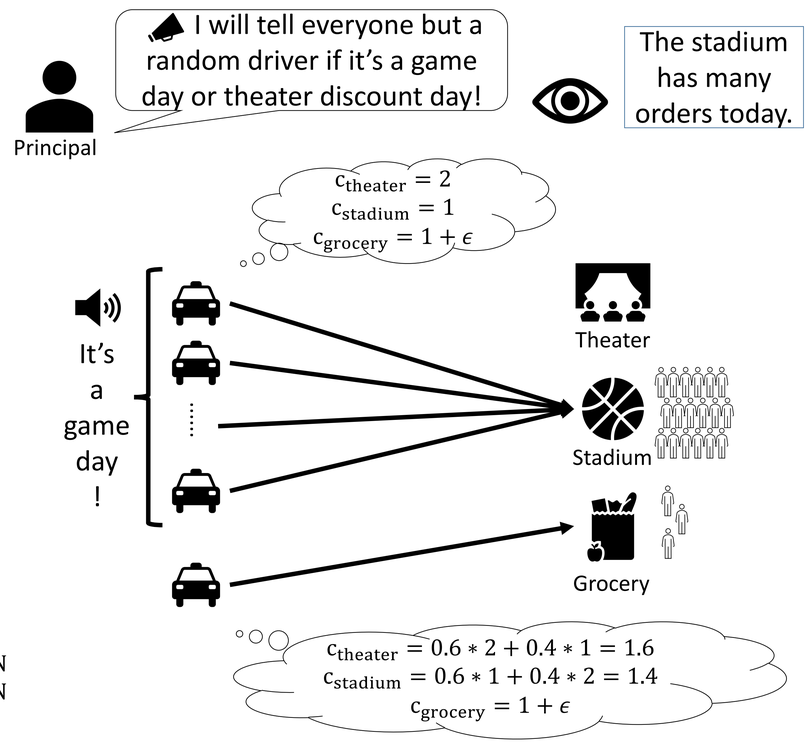
$$\sum_\theta \mu_\theta c^\theta_{a_i^*}(a_i^*, \mathbf{a}^*_{-i}) \le \sum_\theta \mu_\theta c^\theta_{a_i}(a_i, \mathbf{a}^*_{-i})$$

Information Design

Step 1. The principal commits to a signaling scheme π that is publicly known to all agents.

- *Public signaling*: The principal sends the same signal σ to all agents
- *Private signaling*: The principal could possibly send different signals σ_i to each agent i

Step 2. The principal access to the realized state θ



Step 3. Upon receiving signal σ, all agents infer the posterior probability about the state of nature θ and play a singleton congestion game.

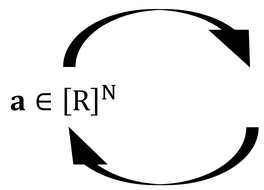
Total congestion is reduced from N to $1 + \epsilon$.

Our Problem: Design the optimal signaling scheme that minimizes total congestion.
Key challenge: The profile of actions chosen by all agents has size $\Omega(R^N)$

The Blessing of Constant Resources

Optimal Public Signaling
Theorem 1. The social-cost minimizing public signaling scheme for SCGs, under optimistic equilibrium selection, can be computed in $\text{poly}(N, 2^{R(R-1)})$ time.

Reduce to polynomial space

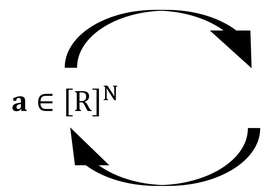


$\mathbf{n} \in [N]^R$: the profile of numbers of agents choosing each resource
$\Lambda \in \{\succeq, \prec\}^{R \times (R-1)}$: agents' preference over resources

Convert back with maximum many-to-one bipartite matching

Optimal Private Signaling
Theorem 2. The social-cost-minimizing private signaling scheme can be computed in $\text{poly}(N^R)$ time.

Reduce to polynomial space



$\{x_{\theta nir}\}$: the marginal probability that conditioned on the state of nature is θ, agent i is assigned to resource r and the configuration of all resources is $\mathbf{n}$.

Convert to signaling scheme π with max flow and flow decomposition

Hardness of SCGs with Many Resources

Equilibrium-Oblivious Intractability of Public Signaling
Theorem 3. It is equilibrium-obliviously NP-hard to obtain a $\left(1 + \frac{1}{5N}\right)$-approximation algorithm for the social-cost minimizing optimal public signaling in SCGs, even when agents have symmetric action sets.

Evidence of Intractability for Optimal Private Signaling
Proposition 1. It is NP-hard to solve the following Optimization Problem (separation oracle of the dual Linear Program), even when all resources have the same linearly increasing congestion functions $c_r(n) = \frac{n}{N}$ and the game is symmetric.

$$\min_{\mathbf{n} \in P(A)} \sum_{r \in [R]} n_r c_r(n_r) - \sum_{r \in [R]} \sum_{i: a_i = r} \sum_{r' \ne r} [c_{r'}(n_{r'} + 1) - c_r(n_r)] \cdot z^i_{r,r'}$$