Random Double Auction: A Robust Bilateral Trading Mechanism

Wanchang Zhang
University of California, San Diego

Design a Trading Mechanism for an Intermediary

- A huge amount of trades are facilitated by intermediaries charging fees for their intermediary services in matching buyers with sellers.
  - Stocks are transacted through a trading platform that typically gets compensation by means of commissions.
  - Cars are transacted through an automobile dealer who charges dealer fees.
  - Many bonds, commodities and derivatives are transacted in the over-the-counter market (OTC) where a market maker earns profits through the bid-ask spread.
- There are many situations in which the uncertainty of the value of the asset being traded is huge, i.e., newly public stock, Tesla's new model.
- How should a profit-maximizing intermediary design trading rules in such situations? Would the intermediary still be able to guarantee a positive profit and thus have strict incentives to offer intermediary services? Maxmin trading mechanism.

Mechanism Design Approach

- When the intermediary has limited information about the value distribution of the buyer and the seller, how should she design a trading mechanism?
  - We assume the intermediary only knows the expectations of of the private values of the buyer and the seller.
  - The profit of the intermediary is the difference between what the buyer pays and what the seller receives.
  - Following the literature on robust mechanism design, the objective is to design a trading mechanism that achieves the maximum profit guarantee. Maxmin trading mechanism.

Model

- Two traders: $i \in I := \{S, B\}$. An asset. An intermediary. S initially holds the asset.
- i's private value: v_i. v_B and v_S may be correlated.
- V_i: the set of possible values of trader i. As a normalization, assume $V_i = [0, 1]$.
- $V = [0, 1]^2$: the set of possible value profiles with a typical value profile $v = (v_B, v_S)$.
- The joint distribution: F.
- The set of all joint distributions on V: ΔV.
- The intermediary only knows the expectations M_B and M_S of the private value of B and S respectively as well as V, but does not know the joint distribution (F) of the values of the traders.
- The set of feasible value distributions:

$$\Pi(M_S, M_B) = \{\pi \in \Delta V : \int v_B d\pi(v) = M_B, \int v_S d\pi(v) = M_S\}.$$

Objective Functions

- The intermediary seeks a dominant-strategy incentive compatible (DSIC) and ex-post individually rational (EPIR) mechanism.
- The set of all DSIC and EPIR mechanisms is denoted as $\mathcal{D}$.
- (q, t_B, t_S): direct mechanism. Trading rule: $q : V \to [0, 1]$. Transfer rule: $t_i : V \to \mathbb{R}$.
- The intermediary looks for a *maxmin trading mechanism*:

$$(q^*, t_B^*, t_S^*) \in \arg\max_{(q,t_B,t_S)\in\mathcal{D}} \min_{\pi\in\Pi(M_S,M_B)} \int_{v\in V} (t_B(v) - t_S(v)) d\pi(v)$$

- $((q^*, t_B^*, t_S^*), \pi^*)$: a saddle point.
- Focus on $M_B > M_S$. A non-trivial pair of expectations.

Random Double Auction

- **Fixed Commission Fee**: The intermediary publicly commits to charging a fixed commission fee $r > 0$ where $r = 1 - \sqrt{1 - (M_B - M_S)}$.
- **Uniformly Random Spread**: The intermediary publicly commits to randomly drawing a spread s uniformly on $[r, 1]$.
- **Midpoint Transaction Price**: The buyer submits a bid price b, and the seller submits an ask price a, simultaneously. If $b - a > s$, then the seller sells the asset to the buyer at the midpoint price $\frac{b+a}{2}$, and each pays the intermediary half of the fixed commission fee $\frac{r}{2}$. Otherwise, no trade takes place, and no one pays or receives anything.

Theorem: The Symmetric Case: $M_B + M_S = 1$

When $M_B + M_S = 1$, **Random Double Auction** is a maxmin trading mechanism. The profit guarantee is $(1 - \sqrt{1 - (M_B - M_S)})^2$.

Miracle 1: Strategy-proofness

- Conditional on trading, a double auction. $\to$ Not strategy-proof.
  - E.g., the buyer has an incentive to submit a bid lower than his true value.
- A random spread decreases this incentive.
  - E.g., if the buyer submits a lower bid, then trade will take place with a lower probability.
- Uniformly random spread makes the mechanism strategy-proof.
  - Buyer's payoff from submitting a bid b when his true value is v_B is $\frac{b-a-r}{1-r} \cdot (v_B - \frac{b+a+r}{2})$.

Miracle 2: "Always" positive profit guarantee

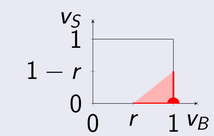
- Uniformly random spread together with the fixed commission fee makes the intermediary indifferent to any feasible value distribution whose support is contained in the set of value profiles where the difference between the buyer's value and the seller's value is higher than r.
  - The profit collected from the bid-ask pair (b, a) if $b - a > r$ is

$$\frac{b - a - r}{1 - r} \cdot r.$$

  - $E[\max\{\frac{v_B - v_S - r}{1-r} \cdot r, 0\}] \geq \max\{E[\frac{v_B - v_S - r}{1-r} \cdot r], 0\} = \max\{\frac{M_B - M_S - r}{1-r} \cdot r, 0\}$. A point mass on (M_B, M_S) yields the profit guarantee.
  - $r = 1 - \sqrt{1 - (M_B - M_S)}$ is optimal.

Miracle 3: Optimal profit guarantee

Symmetric Triangular Value Distribution



- The marginal distribution: A combination of uniform distribution and an atom.
- The conditional distribution: A generalized Pareto distribution with an atom.
- The joint distribution: The support is a symmetric triangular subset of joint valuations. An atom on (1,0).
- Zero-virtual-value condition:

$$\phi(v) = 0, \quad \text{if} \quad v_B - v_S > r \quad \text{and} \quad v \neq (1, 0).$$

- The intermediary is indifferent to any DSIC and EPIR trading mechanism in which 1) trade does not take place if the value profile lies outside the support and trade takes place with probability one when the value of the buyer (resp, the seller) is 1 (resp, 0), and 2) ex-post participation constraints are binding for zero-value buyer and one-value seller.
  - **Random Double Auction** is a best response.

Conclusion

- **Random Double Auction** is novel: a combination of three commonly observed features: random spread+ fixed commission fee + double auction.
- The profit guarantee is positive for any non-trivial pairs of expectations (the expectation of the buyer's value exceeds the expectation of the seller's value).
  - I also characterize maxmin trading mechanisms among *deterministic* DSIC and EPIR mechanisms and find that the profit guarantee is 0 for a range of non-trivial pairs of expectations.